MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

PES0024 – ESSENTIAL STATISTICS

(All Group)

23 OCTOBER 2019
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 6 pages (excluding cover page) with 4 Questions only.
2. Attempt **ALL FOUR** questions.
3. Please provides your solutions in the Answer Booklet provided.
4. **Formula** is provided at the back of the question paper.
5. **Statistical table** is provided at the back of the question paper.

Question 1 (25 marks)

a. A hospital ambulance service handles 0 to 5 service calls on any given day. The probability distribution for the number of service calls is as follows:

Number of service calls, X	Probability
0	0.10
1	0.15
2	0.20
3	0.30
4	0.10
5	0.15

i. Calculate the expected number of service calls (X). (3 marks)
ii. What is the standard deviation in the number of service calls? (6 marks)

b. Consider the following probability density function for a continuous random variable X,

$$f(x)=\begin{cases} tx & ; \quad 3 \le x < 5 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

i. Determine the value of t. (5 marks)
ii. Find $P(x \le 4)$. (5 marks)
iii. Compute the mean of random variable X. (6 marks)

Question 2 (25 marks)

a. In a group of n students, the expected number who wears shoes in class is 3 and the variance is 1.2. Assuming that the number of students who wear shoes follow the binomial distribution.
   i. Find the value of n and p, where p is the probability that a person is wearing shoes when chosen at random. (5 marks)
   ii. Find the probability that less than 2 students in the group wear shoes. (3 marks)

b. The average candidates attend a job interview in a company is 3 candidates per day
   i. Find the probability that less than 3 candidates have come for interview on a given day. (5 marks)
   ii. Calculate the mean and standard deviation of the daily candidates attend a job interview. (3 marks)

Continued …….

c. According to a survey done by MMU research assistant, MMU foundation students spend on average of 4 hours a day at campus to study. Let the daily study time spent for all MMU foundation students have a standard deviation of 0.2846 hours. Find the probability that the daily study time spent will be
   i. greater than 4.20 hours. (4 marks)
   ii. within 0.5 hours of the population mean. (5 marks)

Question 3 (25 marks)

a. The following table gives the monthly salaries (in $1000) of the six officers of a company.

Officer	A	B	C	D	E	F
Salary	8	12	16	20	24	28

   i. Find the population mean and standard deviation. (6 marks)
   ii. Construct a sampling distribution of the mean (without replacement) for sample of size $n = 5$. (8 marks)
   iii. Calculate the sampling error. (6 marks)

b. The heights of a certain population of sugarcane plant follow a normal distribution with mean of 190 cm and a standard deviation of 22 cm. A random sample of 30 plants is chosen, and the mean height is calculated. Find the probability that the sample mean lies between 185 cm and 200 cm. (5 marks)

Question 4 (25 marks)

a. Given the following sample data from a normal population:

10	8	12	15
11	6	5	13

   i. What is the point estimate of the population mean? (3 marks)
   ii. What is the point estimate of the population standard deviation? (4 marks)
   iii. What is the margin error associated with the point estimate of the population mean. (3 marks)
   iv. Construct a 95% confidence interval for the population mean. (4 marks)

Continued ....

b. The manager wants to estimate the average amount a customer spends on lunch from Monday to Friday. A random sample of 115 customers' lunch tabs gave a mean of \$9.74 and a population standard deviation of \$2.93.

i. Find a 99% confidence interval for the corresponding population mean. (4 marks)

ii. Find a 90% confidence interval for the corresponding population mean. (4 marks)

iii. State two methods to increase the width of a confidence interval. Which method is a better alternative? (3 marks)

End of Page.

Formula:

1.

	Mean	Variance
Discrete Random Variable X	$\mu = E(X)$ $= \sum xP(x)$	$Var(X) = E(X^2) - [E(X)]^2$ **where** $E(X^2) = \sum x^2 P(x)$
Continuous Random Variable X	$\mu = E(X)$ $= \int_{-\infty}^{\infty} xf(x)dx$	$Var(X) = E(X^2) - [E(X)]^2$ **where** $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

2.

	Formula	Mean	Standard Deviation
Binomial Probability	$P(x) = \binom{n}{x} p^x q^{n-x}$	$\mu = np$	$\sigma = \sqrt{npq}$
Poisson Probability	$P(x) = \dfrac{e^{-\lambda} \lambda^x}{x!}$	$\mu = \lambda$	$\sigma = \sqrt{\lambda}$

3. The z value for a value of x: $z = \dfrac{x - \mu}{\sigma}$

4. The z value for a value of $\bar{x}$: $z = \dfrac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$

   where $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

5. Point estimate of $\mu = \bar{x}$

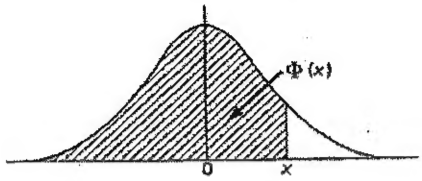
   Margin of error $= \pm 1.96\sigma_{\bar{x}} = \pm 1.96\dfrac{\sigma}{\sqrt{n}}$ or $= \pm 1.96 s_{\bar{x}} = \pm 1.96\dfrac{s}{\sqrt{n}}$

6. The $(1-\alpha)100\%$ confidence interval for μ is $\bar{x} \pm z\dfrac{\sigma}{\sqrt{n}}$

7. Sample variance: $s^2 = \dfrac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

			0·9994		0·99990		0·99995
3·075		3·263		3·731		3·916	
	0·9990		0·9995		0·99991		0·99996
3·105		3·320		3·759		3·976	
	0·9991		0·9996		0·99992		0·99997
3·138		3·389		3·791		4·055	
	0·9992		0·9997		0·99993		0·99998
3·174		3·480		3·826		4·173	
	0·9993		0·9998		0·99994		0·99999
3·215		3·615		3·867		4·417	
	0·9994		0·9999		0·99995		1·00000

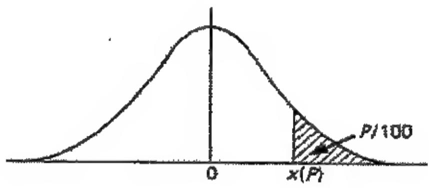
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \dfrac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\dfrac{1}{x^2}+\dfrac{3}{x^4}-\dfrac{15}{x^6}+\dfrac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}} \int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172